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by

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DISCUSSION
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Conditional distributions in the Krugman target zone model and undeclared narrower bands

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Abstract

The paper explicitly derives the conditional distribution of exchange rates and interest rate differentials in the target zone model of Krugman (1991). The exact conditional density function is subsequently utilized in maximum likelihood estimation in which narrower undeclared bands within officially announced target zone limits are allowed for. Estimation results for the four ERM-currencies under consideration reveal that the presence of target zone nonlinearities can not be rejected. The results for the Dutch guilder and the Italian lira thus are in clear contradiction to the findings reported in Ball and Roma (1994) and de Jong (1994) who, however, relied on approximations of the density function. Moreover, the existence of asymmetrically spaced implicit bands can not be rejected for the Belgian and French francs and the Italian lira.

Keywords: Brownian motion, exchange rates, interest rate differentials, probability densities, reflecting barriers, target zones.

JEL classification: F31, F33.

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1 Introduction

This paper examines the conditional distribution of exchange rates and interest rate differentials within a credible target zone arrangement. The analysis is embedded in the canonical target zone model of Krugman (1991). The model of Krugman, notwithstanding its simple structure, has the desirable property that exchange rate jumps at the moment of intervention in the fundamental are precluded. If markets believe that the target zone is credible, intervention at the band limits should not cause discontinuities in the exchange rate as such discontinuities would give rise to arbitrage opportunities. Rational agents then would try to exploit these free lunches by undertaking appropriate actions which then would prevent these discontinuities from arising in the first place.[1] This desirable property renders the Krugman model to be the appropriate starting point for our analysis.

The goals of this paper are twofold. First, conditional distributions within the Krugman model are studied and illustrated. We hereto utilize the exact conditional density function for two-sided regulated Brownian motion that was derived in Veestraeten (2000). Applying this density for the fundamental then permits derivation of the conditional densities of target zone exchange rates and interest rate differentials. Knowledge of the conditional distribution allows a more direct and complete description of the probabilistic character of the evolution of exchange rates and interest rate differentials.

The second main goal of the paper is to employ the conditional exchange rate distribution in an econometric framework. The first novel element relates to the fact that efficient estimation via maximum likelihood now can be pursued via the exact expression for the conditional distribution. Unlike previous research we do not have to rely on approximations of the correct density. For instance, Bekaert and Gray (1998) took account of the restricted domain by assuming a type of truncated normal density for which, however, no clear theoretical underpinning was presented. Ball and Roma (1994) and de Jong (1994) approximated the required density for two-sided reflected Brownian motion by a mixture of densities for one-sided reflection. The second novel element is the extension of the estimation framework towards the potential existence of narrower undeclared zones within the officially announced zones. More in particular, estimation explicitly allows for tighter fluctuation ranges that will be referred to as implicit zones. These implicit zones may be asymmetric in form and thus do not necessarily have to be spaced equidistantly around prevailing central parities. We feel that this is a worthwhile modelling choice as there are ample indications for the presence of

[1] This requirement was first examined in the literature on speculative attacks as the so-called no-expected-arbitrage-profits condition, see Krugman (1979) and Flood and Garber (1991). An extensive discussion and graphical appraisal of its relevance for target zone modelling can be found in Krugman (1991) and Flood and Garber (1991).

such implicit zones. Moreover, this modelling choice has the advantage of being able to assess nonlinearities with respect to actual (implicit) fluctuation limits instead of the wider official limits. Indeed, nonlinearities may be overlooked when inspection is performed with respect to boundaries that in fact may be too wide. Furthermore, the use of approximations for the density, as pursued in the past, may have additionally blurred the picture. Using the exact density and allowing for implicit zones may provide some answer to the question why the theoretically predicted nonlinearities have not been observed in empirical work.

Anticipating the results, we find strong evidence of nonlinearities for the Belgian and French franc. Unlike Ball and Roma (1994) and de Jong (1994), we also detect nonlinearities for the Italian lira and the Dutch guilder. Thus, the use of the exact density and, at the same time, allowing for implicit zones uncover nonlinearities overlooked in previous research. We also find evidence for the presence of narrower implicit fluctuation bands for the two franc currencies and the Italian lira. However, existence of an implicit zone for the Dutch guilder is rejected. This finding is likely to be due to the fact that Dutch monetary authorities heavily relied on intramarginal interventions which questions the appropriateness of the Krugman model as the starting point of our analysis.

The remainder of the paper is organized in four sections. Section 2 reviews the target zone model of Krugman (1991). It first discusses the relationship between fundamentals and exchange rates and then extends the argument to interest rate differentials along the lines suggested in Svensson (1991). Section 3 presents the conditional density function for two-sided regulated Brownian motion for the fundamental. We proceed by specifying the conditional densities for the exchange rate and the interest rate differential. Subsequently, the densities are graphically illustrated. Section 4 presents the econometric application for four currencies that adhered to the Exchange Rate Mechanism (ERM). Estimation focuses on the presence of nonlinearities and implicit (undeclared) target zones. Section 5 concludes and indicates directions for future research.

2 The target zone model of Krugman (1991)

The model of Krugman (1991) proceeded in two steps. The first step specified the dynamics of the fundamental and in the second step fundamentals were linked to the exchange rate via the asset pricing model. Svensson (1991) extended the analysis to the relationship between fundamentals and interest rate differentials. In what follows, we briefly review the functional relationships that, to differing degrees, were presented in Krugman (1991), Svensson (1991) and Delgado and Dumas (1992).

2.1 Target zone exchange rates

Krugman (1991) specified the dynamics of the (logarithm) of the fundamental variable, f, as Brownian motion with drift. Superimposed on this process are two limits between which the fundamental is to evolve. The lower and upper band limits, $\underline{f}$ and $\overline{f}$, reflect the presence of the target zone arrangement. Monetary authorities will decrease (increase) the control variable, i.e. the fundamental, whenever it reaches its upper (lower) band limit. The class of stochastic processes for modelling these features of the fundamental is referred to as the class of regulated or reflected Brownian motion. More specifically, the process is defined as:

$$df = \mu dt + \sigma dz\left(t\right) + dL - dU, \tag{1}$$

where dL and dU are infinitesimal regulators. When the boundaries are reached infinitesimal regulation or instantaneous control, see Harrison and Taksar (1983), pushes the fundamental back into its fluctuation range. The processes L and U are right-continuous, nonnegative and nondecreasing. They only increase, $dL > 0$ and $dU > 0$, when the fundamental hits the lower or upper band limit. In economic terms, L and U then can be interpreted as the cumulative adjustments in (monetary) policy undertaken in support of the fundamental band.[2]

The relationship between the exchange rate and its driving fundamental is specified via the log-linear asset pricing equation. This set-up links the (log of the) exchange rate, s, to the driving fundamental and its own expected change:

$$s = f + \alpha \frac{E[ds]}{dt}, \tag{2}$$

with $\alpha > 0$ and E denoting the expectations operator.[3]

Eqs. (1) and (2) are connected by recognizing that the exchange rate can be expressed as an explicit function of the fundamental or $s = s\left(f\right)$. Assuming $s\left(f\right)$ to be twicely differentiable in f enables application of Itô's lemma. The expectations term in Eq. (2) then can be expressed as:

$$\frac{E[ds\left(f\right)]}{dt} = \mu \frac{ds\left(f\right)}{df} + \frac{1}{2}\sigma^2 \frac{d^2 s\left(f\right)}{df^2}. \tag{3}$$

Inserting Eq. (3) in Eq. (2) gives:

$$s\left(f\right) = f + \alpha\mu \frac{ds\left(f\right)}{df} + \frac{1}{2}\alpha\sigma^2 \frac{d^2 s\left(f\right)}{df^2}.$$

This second-order nonhomogeneous ordinary differential equation has the following solution:

$$s\left(f\right) = f + \alpha\mu + A_1 \exp\left(\lambda_1 f\right) + A_2 \exp\left(\lambda_2 f\right). \tag{4}$$

[2] Flood and Garber (1991) studied discrete interventions or impulse control, see Harrison, Sellke and Taylor (1983). Dumas (1991) presented ample detail on the economic interpretation of different types of control of the driving variable.

[3] Time subscripts are omitted for notational purposes.

The exchange rate solution differs from its counterpart under the free float by the presence of the two exponential terms that cause the renowned bending of the exchange rate function.

The factors λ_1 and λ_2 are the roots of the characteristic equation in λ that solves the homogeneous part of the differential equation:

$$\frac{\alpha\sigma^2}{2}\lambda^2 + \alpha\mu\lambda - 1 = 0,$$

and they equal:

$$\begin{aligned}
\lambda_1 &= \frac{-\alpha\mu + \sqrt{\alpha^2\mu^2 + 2\alpha\sigma^2}}{\alpha\sigma^2} & > 0, \\
\lambda_2 &= \frac{-\alpha\mu - \sqrt{\alpha^2\mu^2 + 2\alpha\sigma^2}}{\alpha\sigma^2} & < 0.
\end{aligned}$$

A_1 and A_2 in Eq. (4) are the constants of integration that will be determined from the boundary conditions for the exchange rate. The exchange rate is function of the fundamental and its boundary behaviour therefore can be obtained from the boundaries applicable to the fundamental. The so-called smooth pasting conditions require the exchange rate function to be flat at the boundaries of the fundamental. Or, the first derivative of $s(f)$ at the limits of the fundamental band has to equal zero:

$$\begin{aligned}
1 + \lambda_1 A_1 \exp\left(\lambda_1 \overline{f}\right) + \lambda_2 A_2 \exp\left(\lambda_2 \overline{f}\right) &= 0, \\
1 + \lambda_1 A_1 \exp\left(\lambda_1 \underline{f}\right) + \lambda_2 A_2 \exp\left(\lambda_2 \underline{f}\right) &= 0.
\end{aligned} \tag{5}$$

The conditions in Eq. (5) then guarantee that intervention in the fundamental does not lead to discontinuities in the exchange rate path. The model thus satisfies the aforementioned no-expected-arbitrage-profits condition.[4]

Eq. (5) allows the constants of integration to be determined as:

$$\begin{aligned}
A_1 &= \frac{\exp\left(\lambda_2 \overline{f}\right) - \exp\left(\lambda_2 \underline{f}\right)}{\lambda_1\left(\exp\left(\lambda_2 \underline{f} + \lambda_1 \overline{f}\right) - \exp\left(\lambda_2 \overline{f} + \lambda_1 \underline{f}\right)\right)}, \\
A_2 &= \frac{\exp\left(\lambda_1 \underline{f}\right) - \exp\left(\lambda_1 \overline{f}\right)}{\lambda_2\left(\exp\left(\lambda_2 \underline{f} + \lambda_1 \overline{f}\right) - \exp\left(\lambda_2 \overline{f} + \lambda_1 \underline{f}\right)\right)}.
\end{aligned}$$

Plugging λ_1, λ_2, A_1 and A_2 into Eq. (4) then fully specifies the exchange rate function given that the fundamental evolves within its fluctuation band. The exchange rates at the upper

[4] As noted earlier, Flood and Garber (1991) examined discrete interventions. The exchange rate in their set-up was prevented from having a discontinuity via a value-matching condition. Taking the limit towards infinitesimal intervention yields the above smooth pasting conditions.

and lower target zone limits are:

$$\begin{aligned} \overline{s} &= s\left(\overline{f}\right) = \overline{f} + \alpha\mu + A_1 \exp\left(\lambda_1 \overline{f}\right) + A_2 \exp\left(\lambda_2 \overline{f}\right), \\ \underline{s} &= s\left(\underline{f}\right) = \underline{f} + \alpha\mu + A_1 \exp\left(\lambda_1 \underline{f}\right) + A_2 \exp\left(\lambda_2 \underline{f}\right). \end{aligned} \tag{6}$$

Having derived the basic functional relationships, we proceed by digressing on two model features that will be useful later. Subsequently, we present an illustration of the functional form of the relation between exchange rates and fundamentals.

The exchange rate is monotonically increasing in the fundamental. This property will be crucial in obtaining the conditional density of the exchange rate in Section 3. Also, announcing a target zone for the fundamental together with the accompanying intervention policy is equivalent to announcing a fluctuation band for the exchange rate. But, the way interventions take place, infinitesimally or discretely, will crucially affect the exchange rate function as pointed out in Flood and Garber (1991) and Froot and Obstfeld (1991).

It can be shown that $\dfrac{ds\left(f\right)}{df}$ is not only larger than or equal to zero but it is also smaller than unity. Hence, movements in the fundamental are transferred to a lower degree into the exchange rate. As such, a credible target zone is inherently stabilizing. Note that this stabilization already occurs prior to actual intervention, cf. the presence of the exponential terms in Eq. (4). This is the so-called target zone honeymoon effect. The exponential terms also lead to the prediction that the honeymoon effect will arise in a nonlinear way. These nonlinearities will now be graphically illustrated and in Section 4 will be at the centre of our estimation exercise.

Figure 1 illustrates the exchange rate function given a fundamental band for which lower and upper limits are set at 3 and 5, respectively. The sensitivity parameter α equals 1.25. In panel (a) the instantaneous standard deviation and drift of the fundamental equal 0.1 and 0.0001. The horizontal axis gives the domain of the fundamental and the corresponding exchange rates are depicted along the vertical axis. The limits of the exchange rate band are given by the two horizontal lines. The exchange rate under the free float is depicted by the (virtually) 45°-line. Nonlinearities in the exchange rate function are hardly noticeable as the small drift and diffusion coefficients make it unlikely that the limits will be reached.

Increasing the volatility parameter to 0.4 in panel (b) clearly produces stronger nonlinearities as the likelihood of intervention increases. The larger prospect of intervention steps up the scope for exchange rate stabilization and mean reversion as rational agents discount the effect of expected intervention into current price setting. Note that the exchange rate band narrowed. This is intuitively explained by the fact that, given the larger degree of dispersion, agents will ex ante expect the exchange rate to be more volatile in the future. But, at the same time the scope for mean reversion and nonlinearities will even more than offset the ex ante expected increase of the fluctuation range.

Insert Figure 1.

The influence of the drift factor can be analyzed by comparing panels (a) and (c) in which μ grows from 0.0001 to 0.1. The increase has an asymmetric affect on nonlinearities as the larger value for μ augments the likelihood of reaching the upper boundary and diminishes expectations that the lower band limit will be attained. The exchange rate band shifts slightly upwards on account of the presence of $\alpha\mu$ in Eq. (6) and the limited role of μ in the nonlinearity factors in that equation.

In conclusion, the scope for nonlinearities in the functional relationship between exchange rates and fundamentals strongly depends on the dynamics of the fundamental. More in particular, mean reversion and stabilization of the exchange rate will be fairly limited for small diffusion and drift coefficients. However, the extent and relevance of nonlinearities crucially hinges on the position of the exchange rate within its fluctuation band.

2.2 Target zone interest rate differentials

Svensson (1991) started from the assumption of uncovered interest rate parity or absence of risk premia. As argued in Svensson (1992) such risk premia are very small and negligible in sufficiently narrow target zones even in the presence of realignment expectations. In view of this result and our assumption of credibility we express the interest rate differential, $\delta = i - i^*$, as:

$$\delta = \frac{E[ds]}{dt}. \tag{7}$$

Plugging Eq. (7) into Eq. (2) and expressing the interest rate differential as a function of the fundamental gives:

$$\delta(f) = \frac{s(f) - f}{\alpha}. \tag{8}$$

Combining Eqs. (4) and (8) specifies the interest rate differential function as:

$$\delta(f) = \frac{\alpha\mu + A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f)}{\alpha}.$$

The derivative of the interest rate differential to the fundamental is:

$$\frac{d\delta(f)}{df} = \frac{\dfrac{ds(f)}{df} - 1}{\alpha}.$$

Thus, the interest rate differential will be monotonically decreasing in the fundamental as the first derivative of the exchange rate is comprised between zero and unity.

As the fundamental evolves between boundaries also the function $\delta(f)$ will be regulated. The boundary interest rate differentials are:

$$\overline{\delta} = \delta\left(\overline{f}\right) = \frac{\alpha\mu + A_1 \exp\left(\lambda_1 \overline{f}\right) + A_2 \exp\left(\lambda_2 \overline{f}\right)}{\alpha},$$

$$\underline{\delta} = \delta\left(\underline{f}\right) = \frac{\alpha\mu + A_1 \exp\left(\lambda_1 \underline{f}\right) + A_2 \exp\left(\lambda_2 \underline{f}\right)}{\alpha},$$

with $\lambda_1, \lambda_2, A_1$ and A_2 as defined above.

Figure 2 depicts interest rate differentials in function of the fundamental. We chose for the parameters adopted earlier in Figure 1. Both the drift and the diffusion coefficients are at low levels in panel (a), namely at 0.0001 and 0.1, respectively. The interest rate differential is nearly linear in the fundamental except for the bending near the edges of the fundamental band. As low drift and volatility reduce the likelihood of intervention, the interest rate differential will therefore over a broad range of the fundamental band equal its counterpart under the free float. The interest rate differential under the free float is given by μ and is represented by the horizontal line. The requirement of intervention at the band limits and the smooth pasting conditions lead to nonlinearities and tangency to the limits of the fundamental band.

The diffusion coefficient is raised to 0.4 in panel (b). Stronger dispersion renders the interest rate differential highly nonlinear. When interpreting the fundamental in terms of the monetary model, for instance, the anticipated movement in the money stock will have the more effect on the interest rate differential the larger the likelihood of intervention becomes. Increasing intervention expectations will, therefore, cause the interest rate differential to deviate strongly from its value under the free float. This rise in intervention needs also explains the widening of the band of the interest rate differential. The decrease in the fluctuation potential for the exchange rate goes at the expense of increasing the domain of the interest rate differential as can be seen from comparing panels (b) in Figures 1 and 2. Decreased exchange rate variability in target zones thus translates in larger interest rate variability.

Insert Figure 2.

Larger drift in the fundamental, i.e. $\mu = 0.1$ in panel (c), strongly increases nonlinearities in the upper part of the band as weakening of the fundamental drives prospects of supporting interventions up. Again, the rise in the drift factor makes the band shift upwards as the likelihood of interventions in the upper part of the fundamental band augments whereas the anticipation of intervention in the lower band segment wanes.

Again, the detection of (important) nonlinearities may be limited to a narrow region of the fundamental band and heavily depends on the dynamic structure of the fundamental.

3 The conditional density of the exchange rate and the interest rate differential

As noted before, the exchange rate and the interest rate differential are one-to-one transforms of the fundamental. Moreover, the first derivatives of the inverted exchange rate and interest rate differential functions to the exchange rate, respectively to the interest rate differential are continuous. These monotone mapping and continuously differentiability properties allow us to express the conditional density of the exchange rate and the interest rate differential in terms of the conditional density of the fundamental through a simple change of variables:[5]

$$p_s(s,t;s_0,t_0) = \frac{p_f\left(s^{-1}(s),t;s^{-1}(s_0),t_0\right)}{\left|\dfrac{ds^{-1}(s)}{df}\right|}, \tag{9}$$

$$p_\delta(\delta,t;\delta_0,t_0) = \frac{p_f\left(\delta^{-1}(\delta),t;\delta^{-1}(\delta_0),t_0\right)}{\left|\dfrac{d\delta^{-1}(\delta)}{df}\right|}, \tag{10}$$

where $p_s(\cdot)$, $p_f(\cdot)$ and $p_\delta(\cdot)$ denote the conditional density functions of the exchange rate, the fundamental and the interest rate differential, respectively. The future and the present point of time are denoted by t and t_0. The functions $s^{-1}(s)$ and $\delta^{-1}(\delta)$ are the inverse functions of $s(f)$ and $\delta(f)$. As the exchange rate (interest rate differential) is monotonically increasing (decreasing) in the fundamental, the functions $s(f)$ and $\delta(f)$ can be (numerically) inverted. Moreover, one can condition the distribution of the interest rate differential on a given intial state of this variable itself, but also on the initial fundamental or initial exchange rate by virtue of the one-to-one relationships between the three variables.

Having specified the change of variables required, Subsection 3.1 will digress on the conditional density function of the fundamental. Subsection 3.2 then graphically illustrates conditional densities for the exchange rate and the interest rate differential.

3.1 The conditional density of the fundamental

Veestraeten (2000) derived the conditional density function for Brownian motion with drift in the presence of two instantaneously reflecting barriers. As the fundamental is assumed to

[5] See Theorem 11 in Mood, Graybill and Boes (1974), p. 200.

follow this type of stochastic process, we can simply re-express his Eq. (11) in our notation which gives:

$$
\begin{aligned}
p_f(f,t;f_0,t_0) = & \sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi(t-t_0)}}\exp\left(\frac{2\mu n\left(\underline{f}-\overline{f}\right)}{\sigma^2}\right)\exp\left(-\frac{\left(f+2n(\overline{f}-\underline{f})-f_0-\mu(t-t_0)\right)^2}{2\sigma^2(t-t_0)}\right)\right\} \\
& +\sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi(t-t_0)}}\exp\left(-\frac{2\mu\left(n\overline{f}-(n+1)\underline{f}+f_0\right)}{\sigma^2}\right)\right. \\
& \qquad \left.\exp\left(-\frac{\left(2n\overline{f}-2(n+1)\underline{f}+f_0+f-\mu(t-t_0)\right)^2}{2\sigma^2(t-t_0)}\right)\right\} \\
& -\frac{2\mu}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\mu\left(n\overline{f}-(n+1)\underline{f}+f\right)}{\sigma^2}\right)\left[1-\Phi\left(\frac{\mu(t-t_0)+2n\overline{f}-2(n+1)\underline{f}+f_0+f}{\sigma\sqrt{t-t_0}}\right)\right]\right\} \\
& +\frac{2\mu}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\mu\left(n\underline{f}-(n+1)\overline{f}+f\right)}{\sigma^2}\right)\Phi\left(\frac{\mu(t-t_0)-2(n+1)\overline{f}+2n\underline{f}+f_0+f}{\sigma\sqrt{t-t_0}}\right)\right\},
\end{aligned}
\tag{11}
$$

where $\Phi(z)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z}\exp\left(-\frac{1}{2}y^2\right)dy$ and denotes the cumulative standard normal distribution function.

Note that the conditional density function consists of four terms that each comprise infinite sums of Gaussian densities and exponentials. Veestraeten (2000) presented simulations that indicate that truncation in the infinite sums can be performed at fairly low levels without endangering accuracy. More in particular, when prediction horizons are small only one term suffices.[6]

3.2 The conditional density of the exchange rate and the interest rate differential

Having obtained the conditional density of the fundamental, we now illustrate densities for target zone exchange rates and interest rate differentials. The densities are calculated by plugging Eq. (11) into Eqs. (9) and (10) and dividing by the absolute value of the first derivative of the inverse function. The figures assume $\underline{f}=3$, $\overline{f}=5$ and $\alpha=1.25$. The time horizon $(t-t_0)$ is set at one year or equals 1.

[6] The econometric application in Section 4 assumes very short prediction horizons of one week or $(t-t_0)=\frac{1}{52}$. However, the truncation point was set at the very safe level of 10. In order to further improve upon accuracy, the cumulative standard normal distribution was calculated using algorithm AS 66 devised by I. D. Hill and reprinted in Griffiths and Hill (1986). This algorithm enjoys higher accuracy than the widely-used algorithms discussed in Zelen and Severo (1964).

Figure 3 depicts the conditional exchange rate density function for $\sigma = 0.1$, 0.2 and 0.3 in panels (a)-(c), respectively. In each of the panels the initial exchange rate is located in the centre of the band and μ is set at 0.02. The horizontal axis represents the exchange rate band. Conditional densities are bell-shaped and peaked around the actual level of the exchange rate. The closer the future exchange rate in panel (a) is located with respect to its limits, the lower conditional densities will be given the relatively low drift and diffusion coefficients and the short time horizon. Increasing the diffusion coefficient augments conditional dispersion and the central region loses probability mass in favour of the margins. More volatile fundamentals squeeze the exchange rate band as noted earlier in the discussion of Figure 1.

Insert Figures 3-4.

Figure 4 portrays conditional densities for the interest rate differential in which σ increases from 0.25, 0.3 to 0.35 when going from panels (a) to (c). The initial state is located in the middle of the band for the interest rate differential and μ equals 0.001. The lower σ the more centred the conditional distribution will be as the interest rate differential becomes less sensitive to the fundamental as noted in the discussion of panels (a) and (b) in Figure 2. Intuitively, low volatility in the fundamental makes it unlikely that bands will be reached. No substantial reaction in the interest rate differential is required as the likelihood of regulation in the fundamental is fairly limited. The centre of the band then collects the bulk of conditional likelihood. Increasing the diffusion coefficient, of course, renders the distribution less peaked. Finally, the domain of the interest rate differential widens with increasing dispersion in the fundamental, whereas for the exchange rate the reverse applied. This again points to the fact that exchange rate stabilization comes at the cost of increasing the fluctuation range of the interest rate differential.[7]

4 Empirical application

This section applies maximum likelihood estimation to the Krugman exchange rate target zone model. Knowledge of the conditional exchange rate density function allows estimation solely on the basis of exchange rate data. Inference upon the nature and the level of the fundamental, that is crucial in for instance structural exchange rate models, is not required.

[7] Svensson (1991) examined instantaneous standard deviations of the exchange rate and the interest rate differential. He showed the existence of a negative trade-off between instantaneous variability in the exchange rate and the interest rate differential.

4.1 Empirical strategy and data

The empirical set-up presents two novel aspects. First, the exact conditional density function is utilized. We thus do not rely on approximations via mixtures of single-barrier densities as in Ball and Roma (1994) and de Jong (1994). Second, we refrain from fixing upper and lower band limits at official levels but estimate them as well. This extends previous estimation frameworks by allowing for implicit narrower bands within wider official target zones. Also, we allow these bands to be asymmetrically spaced around parities. This is a potentially rewarding feature as for instance the Belgian franc during the years 1987 to 1989 fluctuated without much volatility in the upper part of its bilateral band vis-à-vis the German mark. This modelling choice also allows examination of nonlinearities with respect to (asymmetric) implicit bands. In doing so, the potential danger of overlooking nonlinearities through working with unnecessarily wide zones can be circumvented. As noted earlier when discussing Figure 1, such a strategy may be desirable due to the fact that occurrence of nonlinearities strongly depends on the position of the exchange rate within its (true) fluctuation band.

Exchange rates represent the log of Wednesday prices for the market in London. We choose for weekly data in order to capture one of the main forecasting horizons in the foreign exchange market. The data are obtained from Datastream. The sample consists of the Belgian franc, the Dutch guilder, the French franc and the Italian lira each quoted against the German mark as the price of one German mark. The sample for the first three currencies spans the period March 1994 until February 1997 (153 observations). The starting point is set in function of the prerequisite of credibility as the ERM-turmoil of 1993 in the case of the franc currencies only faded away towards the end of that year. The endpoint reflects doubts on the appropriateness of the target zone model in view of the prospect of EMU. De Grauwe, Dewachter and Veestraeten (1999) argued that the start of EMU on January 1 1999 had to be seen as a regime switch that already months earlier drastically changed exchange rate dynamics. More in particular, the role of fundamentals in pricing was reduced in favour of the conversion rate. The applicability of a fundamental-driven model thus dwindled in the course of 1997-1998. Official band limits for the two franc currencies were $\pm 15\%$, whereas Dutch monetary authorities explicitly opted for the former $\pm 2.25\%$ zone. Data for the Italian lira span the period between February 1987 and December 1989 (152 observations). This period followed the devaluation of the lira on January 12 1987 and was characterized by a sharp reduction in realignment expectations as noted in for instance Svensson (1993) and Rose and Svensson (1994). The sample ended in December 1989 in view of excluding the (technical) devaluation that accompanied narrowing of the fluctuation band from $\pm 6\%$ to $\pm 2.25\%$ on January 8 1990.

4.2 The likelihood function

Maximization of the likelihood function requires knowledge of the joint density of observed exchange rates.[8] The strong Markov property of reflected Brownian motion allows complete conditioning of the conditional density of the fundamental upon its actual state. Thus, the history of the fundamental is not relevant in making predictions. Obviously, the Markov property carries over to the exchange rate given the one-to-one relationship between fundamentals and exchange rates. The joint distribution of $\mathbf{s}$, $\mathbf{P}(\mathbf{s})$, then can be written as:

$$\mathbf{P}(\mathbf{s}) = \prod_{t=0}^{T-1} p_s\left(s_{t+1}, t+1; s_t, t\right).$$

The loglikelihood function for the observed exchange rate series $\mathbf{s}$ is defined in terms of the parameter vector $\boldsymbol{\Theta} = \left\{\alpha,\ \mu,\ \sigma,\ \underline{f},\ \overline{f}\right\}$:

$$LogLikF(\mathbf{s}, \boldsymbol{\Theta}) = \sum_{t=0}^{T-1} \ln\left(p_s\left(s_{t+1}, t+1; s_t, t\right) \mid \boldsymbol{\Theta}\right).$$

The parameters in the maximization process consist of the four parameters that fully specify the stochastic process of the fundamental, namely the instantaneous drift and standard deviation, μ and σ, and the upper and lower band limits, $\overline{f}$ and $\underline{f}$. The fifth parameter is the sensitivity of the exchange rate to its own expected change, α.

Earlier research showed that application of maximum likelihood in the context of target zone models is severely hampered by the presence of multicollinearity between α and σ.[9] As illustrated in Figure 1 in Ball and Roma (1994), the likelihood surface is very flat in the parameter pair (α, σ). We also encountered this problem and therefore estimated the model for an array of different starting values for the parameter α. The second effect of the flatness of the likelihood surface relates to the calculation of standard errors. Obtaining standard errors for all five coefficients is not sensible as the curvature of the surface is insufficient as noted in Ball and Roma (1994) and de Jong (1994). We therefore calculate asymptotic standard errors for four parameters and keep α at its estimated value. Model specification will be formally tested by performing two Likelihood Ratio (LR) tests.

4.3 Estimation results

Table 1 presents estimation results. The sensitivity parameter α governs the presence of nonlinearities as it determines the extent in which expectations (of target zone interventions)

[8] Note that the dependent variable in Ball and Roma (1994) and de Jong (1994) was the (log of the) position of the exchange rate with respect to the central parity. This choice reflected their attention to official symmetric bands.

[9] This problem, however, also arose within the Method of Simulated Moments as indicated by Lindberg and Söderlind (1994a,b).

affect current exchange rate pricing, see Eq. (2). Large values for α imply that bands play a significant role in pricing and thus cause nonlinearities and the target zone honeymoon effect. The parameter α is estimated at 2.13 in the case of the Belgian franc which seems to point to the presence of (strong) nonlinearities. This holds even stronger for the French currency for which the point estimate amounted to 4.34.

The Dutch guilder presents a different picture. The point estimate for α is close to zero (0.01) which at first sight would induce us to conclude that nonlinearities for the Dutch currency are absent. The parameter α is estimated at 0.31 in the case of the Italian lira. The lira thus takes an intermediate position between the franc currencies and the guilder. Whether or not α is significantly different from zero and equivalently whether or not nonlinearities are present will be discussed below via an LR-test.

Insert Table 1.

The third and fourth columns give the estimated lower and upper limits of the fundamental band.[10] These estimates together with the estimates for α, μ and σ allow us to calculate the corresponding (implicit) exchange rate band limits. This is illustrated in Table 2.

The first and second columns in Table 2 represent the implied lower and upper exchange rate band limits. The third and fourth columns present the band width in relation to the prevailing central parity.

Insert Table 2.

The upper and lower band widths, in absolute value, are clearly differing from each other and differ strongly from official levels for the franc currencies. These currencies were thus subject to undeclared narrower bands that were asymmetric in nature.[11] Estimated widths for the guilder are extremely close to the official limits of $\pm 2.25\%$. Results for the Italian lira suggest the existence of a narrower and asymmetric band within the official $\pm 6\%$-band.

We now turn to two model specification tests. The first LR-test sheds more light on the issue whether α significantly differs from zero, i.e. we formally test for the presence of nonlinearities in the four exchange rate series. Constraining α to zero causes s to equal f as can be seen from Eq. (2). In other words, the stochastic process of the fundamental carries over to the exchange rate. The exchange rate in the constrained model thus follows regulated

[10] The fact that α is close to zero in the case of the Dutch currency refrained the information matrix from being positive definite. The near-zero value of α implies that the influence of expectations and thus of band limits is virtually absent which creates huge standard errors for the estimated band limits. For the guilder, we therefore chose to fix α, $\underline{f}$ and $\overline{f}$ in order to be able to calculate standard errors for the remaining two coefficients.

[11] The second LR-test examines whether implied band limits significantly differ from official limits.

Brownian motion where, as before, we include the boundaries in the parameter set. This brings us to the distribution of the test statistic. The restriction on α is non-standard as α is restricted to the positive half-line and the restriction lies on the boundary of its domain. Chernoff (1954) showed that the resulting LR-statistic is distributed $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$. We follow de Jong (1994) and approximate this distribution as χ_1^2. The test is therefore conservative.

The first and second columns in Table 3 give the test statistic and the p-value. The null of absence of nonlinearities can safely be rejected for the two franc currencies in line with the conclusions reported in de Jong (1994) and Ball and Roma (1994).

In the case of the guilder, the null hypothesis of $\alpha = 0$ or absence of nonlinearities is also to be rejected. Thus, notwithstanding the fact that the point estimate of α is extremely small, its size appears to be sufficient to generate nonlinearities. This finding for the guilder is in clear contradiction to the results reported in Ball and Roma (1994) and de Jong (1994) who were unable to reject the null hypothesis of no nonlinearities. The difference in conclusion is likely to be due to the nature of the conditional density function utilized in estimation. This paper implemented the exact conditional density function for two-sided reflection whereas Ball and Roma (1994) and de Jong (1994) had to rely on approximations based on densities for one-sided reflection.

Results for the Italian lira reveal that also this currency is characterized by the presence of nonlinearities. This is in clear contrast to de Jong (1994) who concluded that the null could not be rejected. He explained his finding by arguing that the lira did not use the entire $\pm 6\%$ fluctuation range. Remember that his empirical model was built up around official boundaries and as such may have biased results towards not being able to reject the null hypothesis. Indeed, examination of nonlinearities with respect to bands that are too wide may overlook their presence as nonlinearities are essentially limited to the regions near the true limits. Our framework thus indicates that nonlinearities are clearly present when they are gauged with respect to implicit narrower zones.

Insert Table 3.

In sum, nonlinearities are detected for the four currencies under investigation. Even the guilder mark exchange rate appears to be characterized by small but significant nonlinearities. This findings points to the relevance of using the exact conditional density function. In addition, allowing for implicit zones may be seen as a second safeguard against overlooking nonlinearities as exemplified by the lira.

The second specification test, LR-test 2 in Table 3, inquires into the question whether estimated implicit bands significantly differ from officially announced bands. We thus formally test for the presence of implicit zones. The restricted model fixes band limits for the fundamental at levels that correspond to announced exchange rate band limits given the point estimates for the triple (α, μ, σ) as reported in Table 1. The band limits are then obtained by solving the system of nonlinear equations in Eq. (6). We have two standard restrictions and the test statistic is thus distributed as χ^2_2.

Results are given in the third and fourth columns. The null hypothesis of no implicit zones can safely be rejected for the two franc currencies. Or, they evolve between narrower implicit bands. This is hardly surprising as these two currencies by no means used up the official $\pm 15\%$ band.

The existence of a narrower zone is to be rejected for the guilder. This result is at first sight surprising as most accounts point to the presence of such a narrower zone for the Dutch guilder. Our counterintuitive finding is linked to the way Dutch monetary authorities maintained the implicit zone. The guilder was kept in a narrower zone by means of mean-reverting interventions. In other words, the narrower zone was operated via intramarginal interventions and not by marginal interventions as assumed in the Krugman model. For this country, the Krugman model is of limited applicability and should be replaced by a reflected Ornstein-Uhlenbeck structure that can cope with (strong) mean-reverting fundamentals. Cavaliere (1998) used a different strategy in examining the presence of implicit target zones as his test statistic was not embedded within a specific target zone model. More precisely, it was built upon the autocorrelation function of the exchange rate that was assumed to follow reflected Brownian motion. Cavaliere (1998) found strong evidence for a narrower undeclared zone in line with accounts for the guilder.

For the Italian currency we can also safely conclude that the estimated implicit exchange rate zone differed from the official one. Thus, Italian monetary authorities did apply a narrower undeclared zone as suggested by de Jong (1994), see before. However, this contradicts the finding reported by Cavaliere (1998) who examined the lira over the same sample period but failed to detect an undeclared target zone. Responsible for this difference in conclusion is the fact that, within our methodology, nonlinearities and (implicit) band limits can co-exist. The test statistic of Cavaliere (1998) can not account for nonlinearities as it is built around a stochastic framework that is essentially linear, cf. the absence of target zone elements such as the smooth pasting conditions.

The second LR-test allows us to accept the presence of narrower undeclared bands for the Belgian, French and Italian currencies. The test refuted the presence of an implicit zone for the guilder which points to the limited applicability of the Krugman model for that currency as the guilder was clearly kept within narrower bands.

Figure 5 visualizes estimation results for the four currencies. The horizontal axis in each of the panels specifies the estimated fluctuation range for the log fundamental and the log exchange rate is depicted along the vertical axis. The exchange rate band limits are given by the horizontal lines that designate (in descending) order the official upper limit, the implied upper limit, the implied lower limit and the official lower limit. The immense distance between official and implicit limits in the case of the Belgian franc would have completely blurred the picture for that currency. As a result, we only depict the estimated narrower bands. The solid line between the two implicit boundaries portrays the relationship between fundamentals and exchange rates calculated on the basis of the estimation results presented in Table 1. Superimposed on this relationship is the scatter plot of observed exchange rates and their corresponding values for the fundamental.

Insert Figure 5.

The estimated relationship between fundamentals and exchange rates for the Belgian franc in panel (a) is clearly nonlinear and evidences the predicted S-shape. The dots are heavily concentrated in the lower part of the estimated band. As nonlinearities typically occur close to band limits, the finding of a substantial value for α then should come as no surprise. In other words, nonlinearities appear to be a clear characteristic of the exchange rate path for the Belgian currency. The same conclusion holds for the French franc in panel (b).

Panel (c) presents the picture for the Dutch guilder. From the above point estimate we know that nonlinearities in the exchange rate are small but significant. The near-zero value for α thus implies that the relationship between exchange rates and fundamentals is virtually linear. The figure also shows the surprising result that the implied exchange rate band virtually coincides with the official band (in fact the lines for the upper (lower) limits of the implicit and official bands overlap).

Finally, the Italian lira in panel (d) also reveals nonlinearities when performing estimation with respect to potentially narrower bands.

5 Conclusion

This paper first examines the conditional exchange rate and interest rate differential distributions within a credible target zone arrangement. The starting point is the canonical target zone model of Krugman (1991) in which the fundamental was assumed to follow a continuous-time random walk between two reflecting barriers. We describe the functional relationship between the control variable, i.e. the fundamental, and the exchange rate and the interest rate differential.

First, we applied a change of variable that allows to express the conditional density of target zone exchange rates and interest rate differentials in terms of the conditional density of the fundamental for which we used the exact expression derived in Veestraeten (2000). Conditional densities are shown to be bell-shaped and peaked around the present value of the interest rate differential or the exchange rate, albeit that distributions for interest rate differentials are generally more concentrated.

The second main focus of this paper was directed towards maximum likelihood estimation. The empirical framework extended existing literature in two ways. First, the exact conditional density function was used in contrast to Ball and Roma (1994) and de Jong (1994) who approximated the density. Second, the estimation set-up allowed for undeclared narrower or implicit bands within officially announced bands. These implicit bands, moreover, could be asymmetrically spaced around parities. These extensions are worthwhile for two reasons. First, implicit zones have always been integral part of target zone arrangements such as the ERM. The second and main reason is related to the search for nonlinearities in exchange rates. Estimating with respect to official band limits when actually narrower zones are pursued may overlook nonlinearities as the boundaries in estimation are unnecessarily wide. The intuition is simple. Nonlinearities typically arise near the fluctuation limits, so their presence may become illusory when inadequate limits are implemented in estimation. Moreover, using the exact conditional density may overcome the potential bias implied by using approximate densities.

The empirical application is pursued for four ERM-currencies, in casu the Dutch guilder, the Belgian franc, the French franc and the Italian lira, during sample periods for which credibility could safely be assumed. In line with existing research, the presence of nonlinearities is accepted for the two franc currencies. In contrast to Ball and Roma (1994) and de Jong (1994), small but significant nonlinearities are also detected for the Dutch guilder. Estimation with respect to narrower zones using the exact density formulation appears to uncover nonlinearities that were overlooked in the past. Contrary to de Jong (1994) nonlinearities are also detected for the Italian lira. For three of the four currencies, the existence of implicit narrower target zones can safely be accepted which in the case of the Italian currency contradicts the finding of Cavaliere (1998). However, implicit zones are not found for the Dutch currency in sharp contrast to intuition and the results reported in Cavaliere (1998). This is likely to be due to the fact that our framework is built upon the target zone model of Krugman (1991) that only allows for marginal interventions. As Dutch monetary authorities heavily engaged in intramarginal interventions, a reflected Ornstein-Uhlenbeck set-up would have been more appropriate for this currency.

The Krugman model is based on the strong assumption of coupling perfect credibility with the sole use of marginal interventions. The approach of this paper, however, can be extended to the so-called second generation target zone models that introduce (stochastic) devaluation risk and intramarginal interventions. Ball and Roma (1994) also present estimations under the assumption of the fundamental following a reflected Ornstein-Uhlenbeck process, i.e. allowed for intramarginal interventions. The model and estimation framework can also be extended towards the inclusion of imperfect credibility, i.e. of introducing devaluation expectations. Various routes for future research are open to this end. One can opt for bivariate processes as for instance pursued in Ball and Roma (1993) who superimpose a Poisson state-dependent jump upon a diffusion process. Bertola and Svensson (1993) introduced the notion of a composite fundamental in which stochastic devaluation risk is included and assumed this composite fundamental to follow reflected Brownian motion. Lindberg and Söderlind (1994b) specified the stochastic process for the composite fundamental as an Ornstein-Uhlenbeck process and as such combined mean-reverting fundamentals with imperfect credibility. The approach of this paper can also be seen as a first step towards modelling of (credible) crawling band arrangements. Introduction of moving, time-dependent, bands could be pursued by imposing linear retaining or straight line barriers, see Park and Schuurmann (1980), Gerber, Goovaerts and De Pril (1981), Park and Beekman (1983) and Teunen and Goovaerts (1994).

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Table 1: **Maximum likelihood estimation vis-à-vis the German mark.**

	α	μ	σ	$\underline{f}$	$\overline{f}$	Loglik. value
Belgian franc[1,3]	2.12936 (na)	−0.03305 (0.0354)	0.03366 (0.0162)	3.00805 (0.0031)	3.09838 (0.0830)	863.06569
French franc[1]	4.34464 (na)	−0.03984 (0.0569)	0.06546 (0.0343)	1.16723 (0.0128)	1.46743 (0.0908)	649.40657
Dutch guilder[1]	0.01035 (na)	0.00019 (0.0024)	0.00394 (0.0002)	0.09679 (na)	0.14186 (na)	929.09911
Italian lira[2]	0.31062 (na)	0.01840 (0.0187)	0.03319 (0.0034)	6.54870 (0.0031)	6.64785 (0.0255)	615.67898

[1] Sample: 94.03.02-97.01.29 (153 observations).
[2] Sample: 87.02.04-89.12.27 (152 observations).
[3] Asymptotic standard errors are given within parentheses.

Table 2: **Implied exchange rate band width vis-à-vis the German mark.**

	$\underline{S}$	$\overline{S}$	lower band (%)	upper band (%)
Belgian franc[1]	20.53635	21.06722	−0.43410	+2.14163
French franc[1]	3.35386	3.72797	−0	+11.15474
Dutch guilder[1]	1.10195	1.15209	−2.20015	+2.24985
Italian lira[2]	709.73296	763.04393	−1.54509	+5.87554

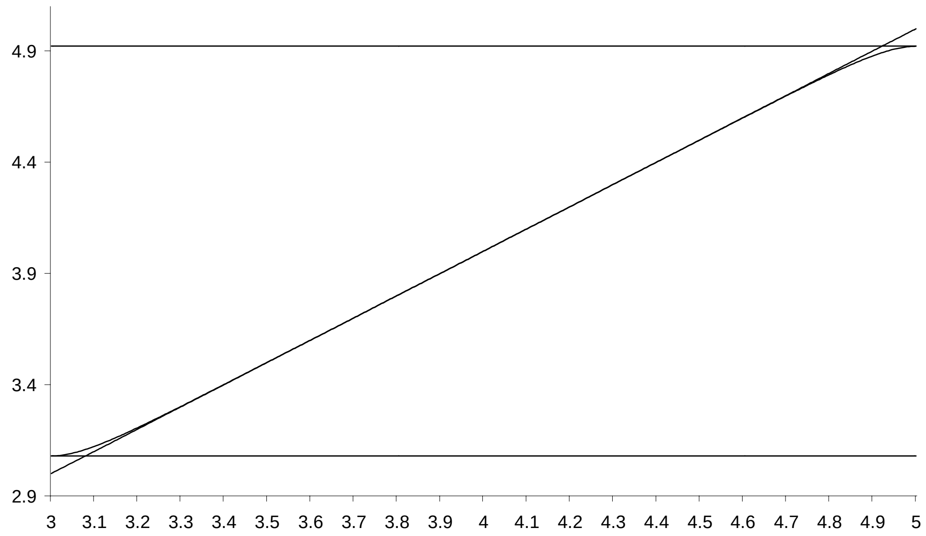
Notes: see table 1.

Table 3: **Likelihood Ratio tests.**

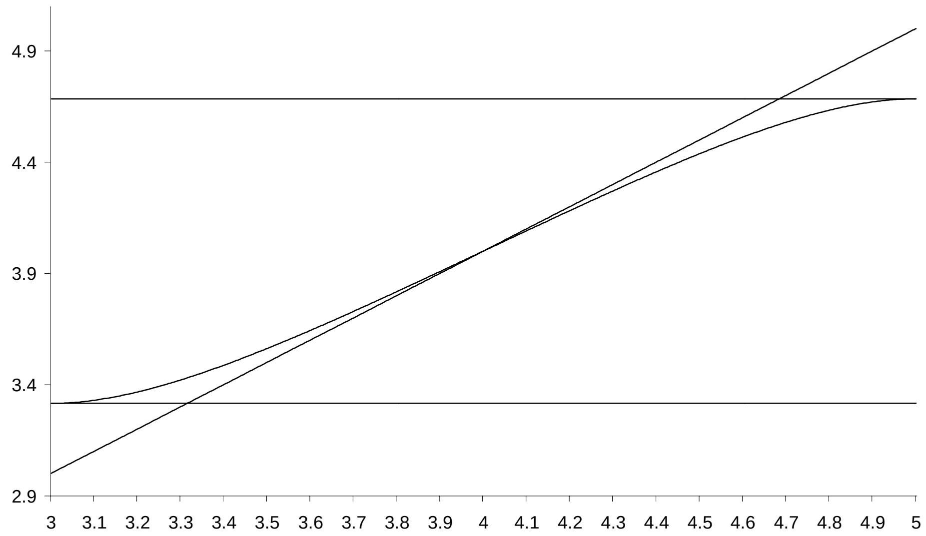
	LR-test 1	p-value[1]	LR-test 2	p-value[2]
Belgian franc	70.48436	4.6E-17	70.48434	4.9E-16
French franc	50.85644	9.9E-13	46.33714	8.7E-11
Dutch guilder	6.14162	0.01320	3.0E-7	0.99999
Italian lira	7.43302	0.00640	7.55428	0.02289

[1] Under χ^2_1.
[2] Under χ^2_2.

Figure 1: The relationship between the exchange rate and the fundamental in a target zone and under the free float



Panel (a)



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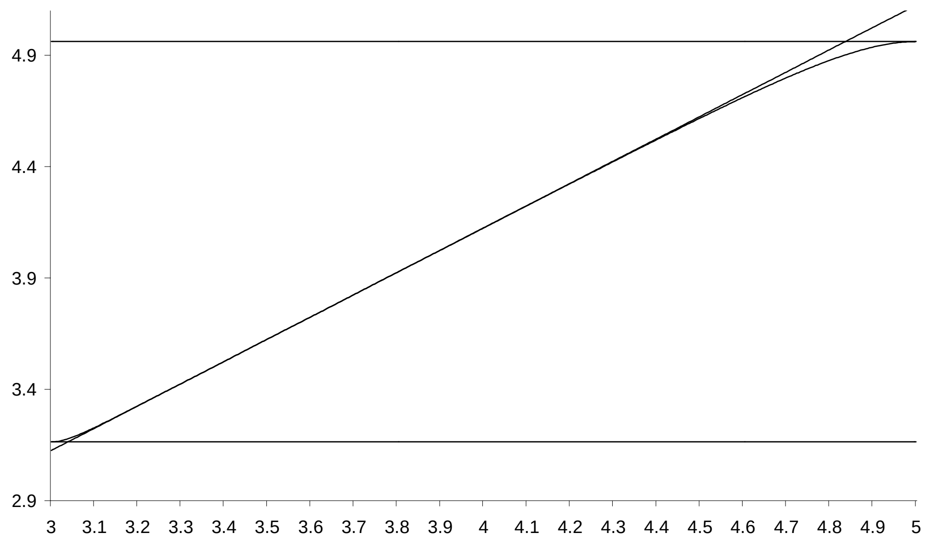
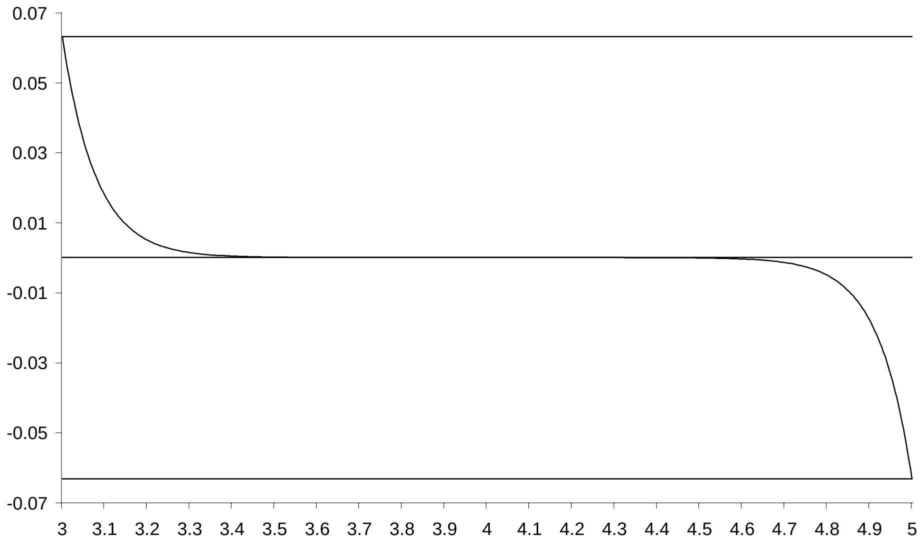


Panel (c)

Figure 2: The relationship between the interest rate differential and the fundamental in a target zone and under the free float



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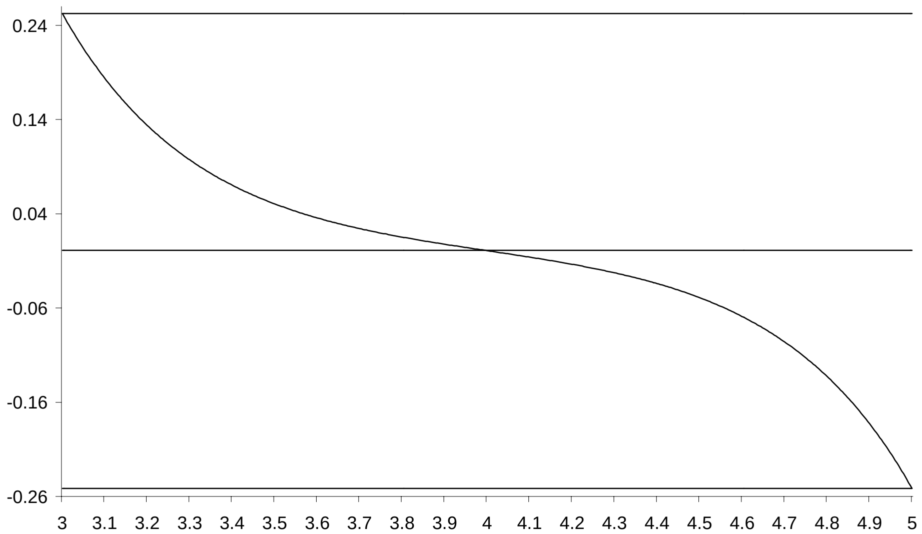

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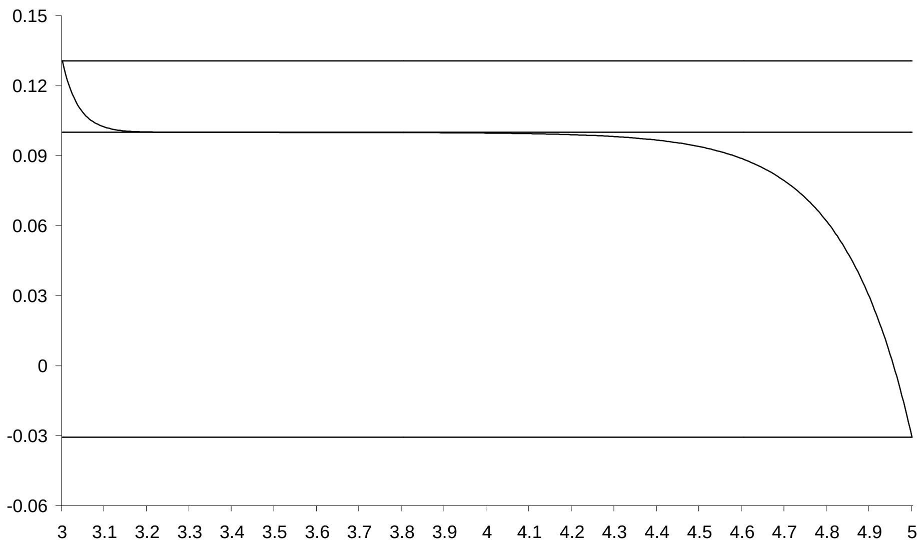
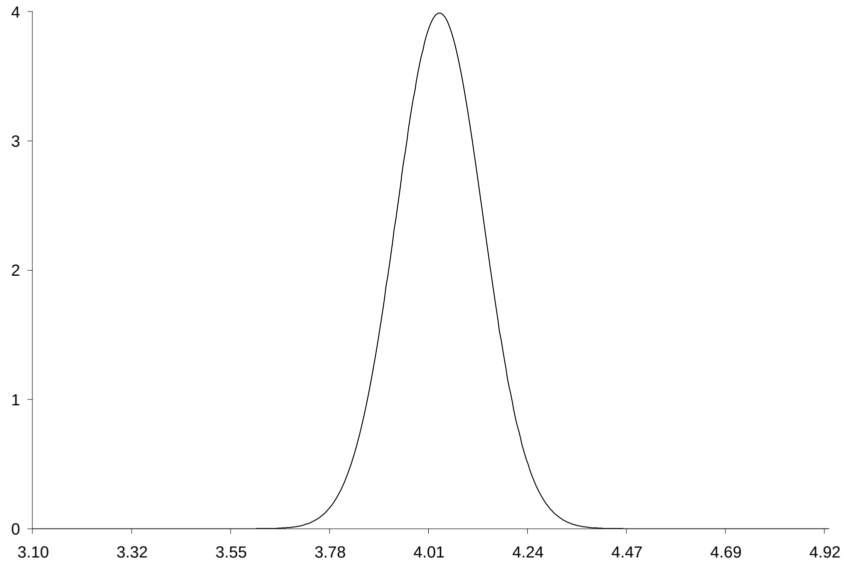
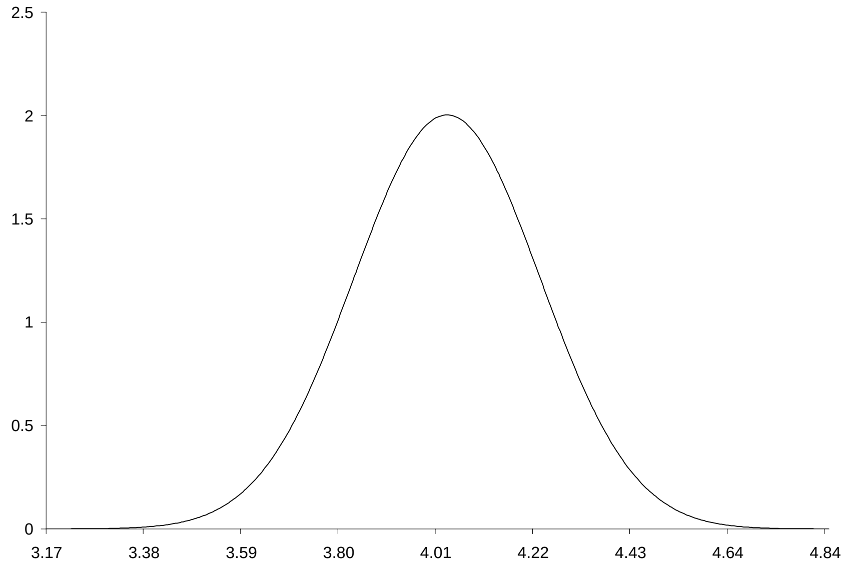


Panel (c)

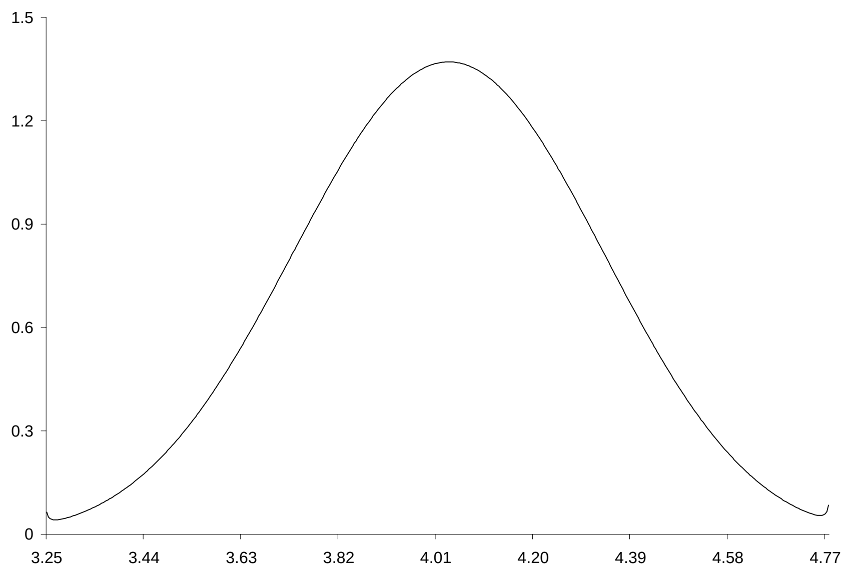
Figure 3: Conditional density of the exchange rate for three diffusion coefficients.



Panel (a)

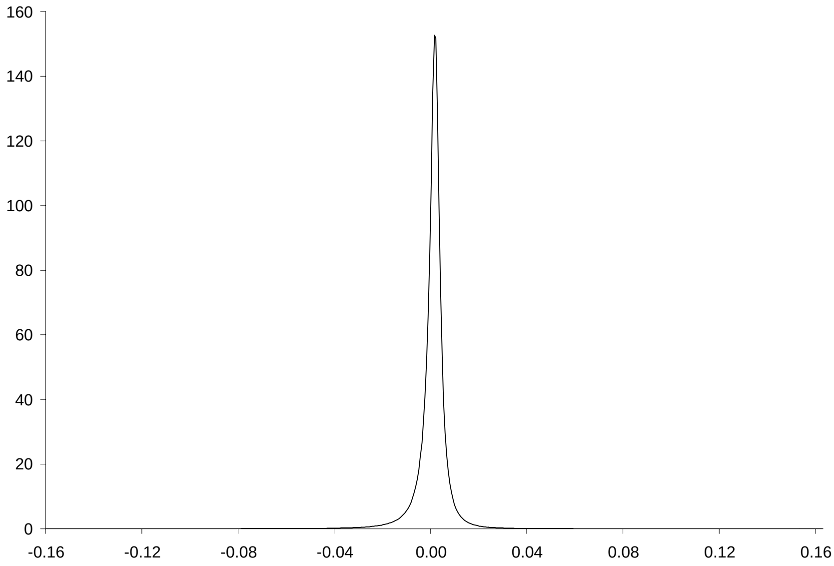


Panel (b)

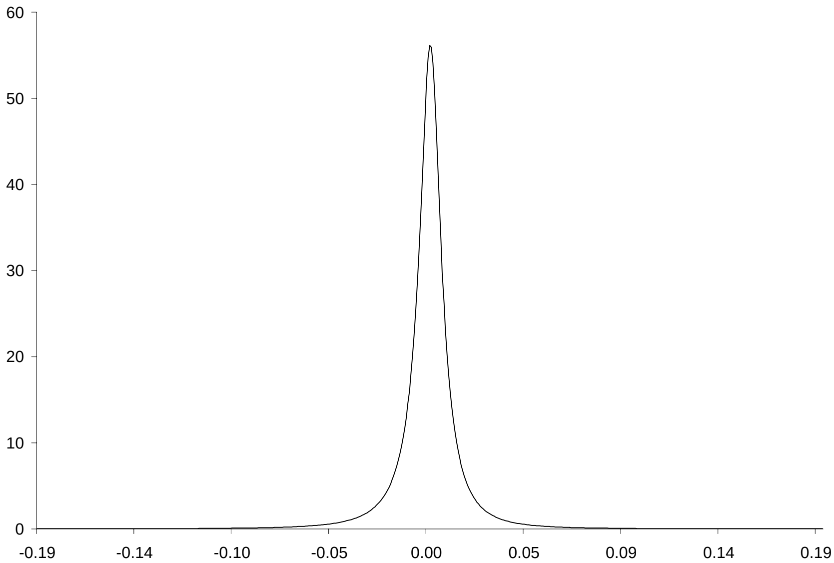


Panel (c)

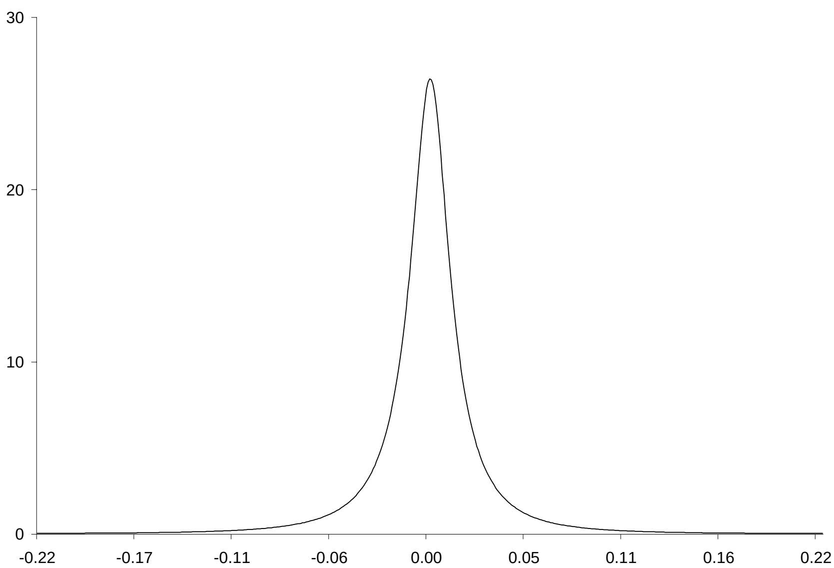
Figure 4: Conditional density of the interest rate differential for three diffusion coefficients.



Panel (a)



Panel (b)



Panel (c)

Figure 5: Relationship between fundamentals and exchange rates for the Belgian franc (panel a), the French franc (panel b), the Dutch guilder (panel c) and the Italian lira (panel d).